Applications of Optimization Techniques in Active Sensing and Communication Systems

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Real Worlds Optimization Projects

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Outline

- Introduction
- A radar example
  - Problem formulation
  - The algorithm
- An example for communication systems
  - Problem formulation
  - The algorithm
- Summary and conclusion

Introduction

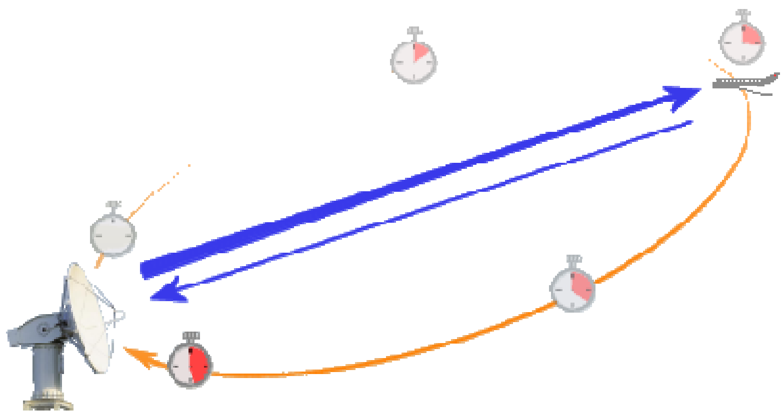
- Active sensing systems: radars, sonars, etc.
  - Radar: **RA**dio **D**etection **A**nd **R**anging

- Working principle: transmission of waveforms toward medium and process the received echo

- Actively sensing the medium
    - Transmission of signal

Introduction

- Being active: waveform transmission

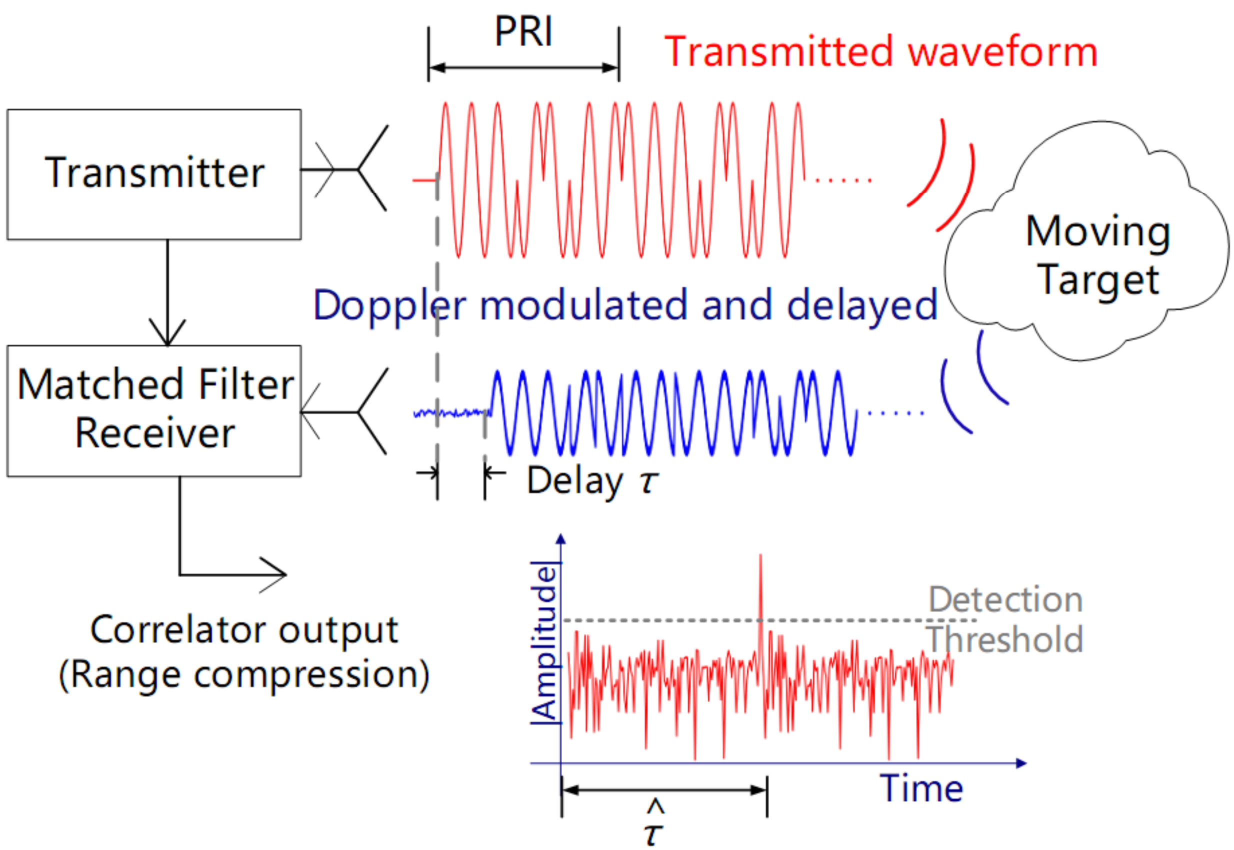


http://www.radartutorial.eu/01.basics/Radar%20Principle.en.html

- The system performance & transmit signal

Introduction

- A simplified block diagram

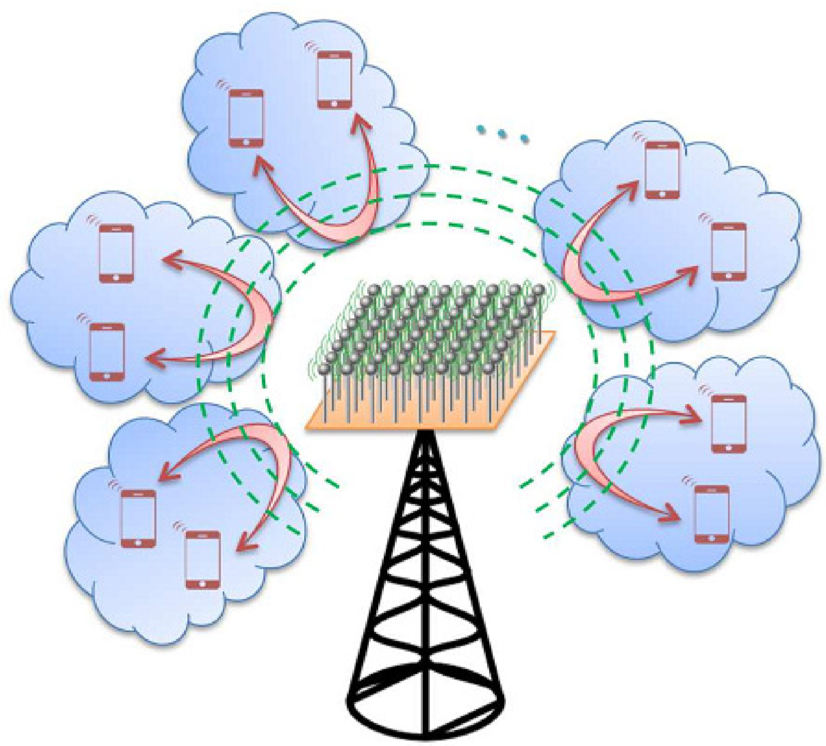


Introduction

- Optimize Tx-Rx pair to improve performance
  - Tx transmit waveform
  - Rx filter
  - Various performance metrics
    - Signal-to-noise-plus-interference (SINR)

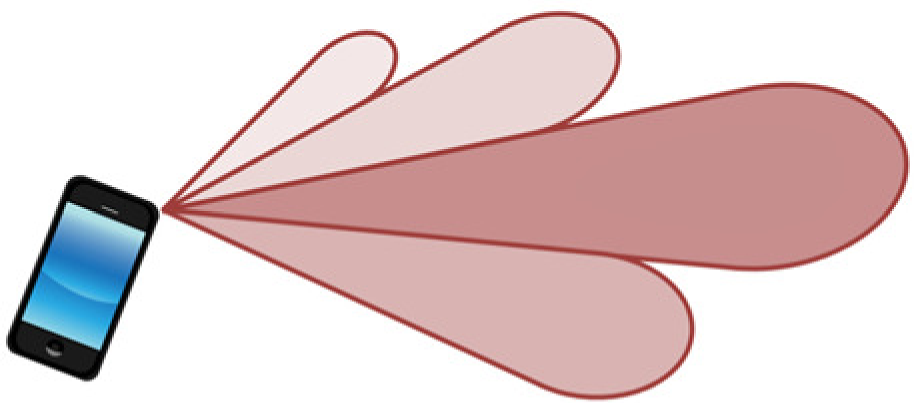
Introduction

- Communication Systems
  - Users communicate with themselves via Base stations, Relays, etc



Introduction

- Optimize Tx-Rx to improve performance
  - User, relay, base station
  - Metrics: user achievable rate



A radar example

- Design Tx transmit signal and Rx filter
  - Improve SINR at the filter output
- Main challenges
  - Clutter: Signal-dependent interference
    - Echo of unwanted targets like mountain, trees, etc.
  - The target speed is unknown
  - Doppler phenomena

$$\mathbf{r} = \underbrace{\alpha_T \mathbf{x} \odot \mathbf{p}(\nu)}_{\text{target}} + \underbrace{\mathbf{c} + \mathbf{n}}_{\text{Clutter+interference}},$$

A radar example

- SINR expression for Doppler \nu

$$\mathrm{SINR}(\nu) = \frac{|\alpha_T|^2 \left| \mathbf{w}^H \left(\mathbf{x} \odot \mathbf{p}(\nu)\right) \right|^2}{\mathbf{w}^H \boldsymbol{\Sigma}_{\mathbf{c}}\left(\mathbf{x}\right) \mathbf{w} + \mathbf{w}^H \mathbf{M} \mathbf{w}}$$

clutter

interference

  – Clutter covariance:

$$\boldsymbol{\Sigma}_{\mathbf{c}}\left(\mathbf{x}\right) = \sum_{k=0}^{N_c-1} \sum_{i=0}^{L-1} \sigma^2_{(k,i)} \mathbf{J}_k \boldsymbol{\Gamma}(\mathbf{x}, (k,i)) \mathbf{J}_k^T$$

A radar example

- The design problem: a maxmin
  - Robustifying w.r.t to unknown Doppler

$$\mathcal{P}\begin{cases} \max\limits_{\mathbf{x},\mathbf{w}} \ \min\limits_{\nu} & \dfrac{\left|\mathbf{w}^H\left(\mathbf{x}\odot\mathbf{p}(\nu)\right)\right|^2}{\mathbf{w}^H\boldsymbol{\Sigma}_{\mathbf{c}}\left(\mathbf{x}\right)\mathbf{w}+\mathbf{w}^H\mathbf{M}\mathbf{w}} \\ \text{subject to} & \|\mathbf{x}\|^2 = e \\ & \|\mathbf{x}-\mathbf{x}_0\|^2 \leq \delta \\ & \nu \in \Omega \end{cases}$$

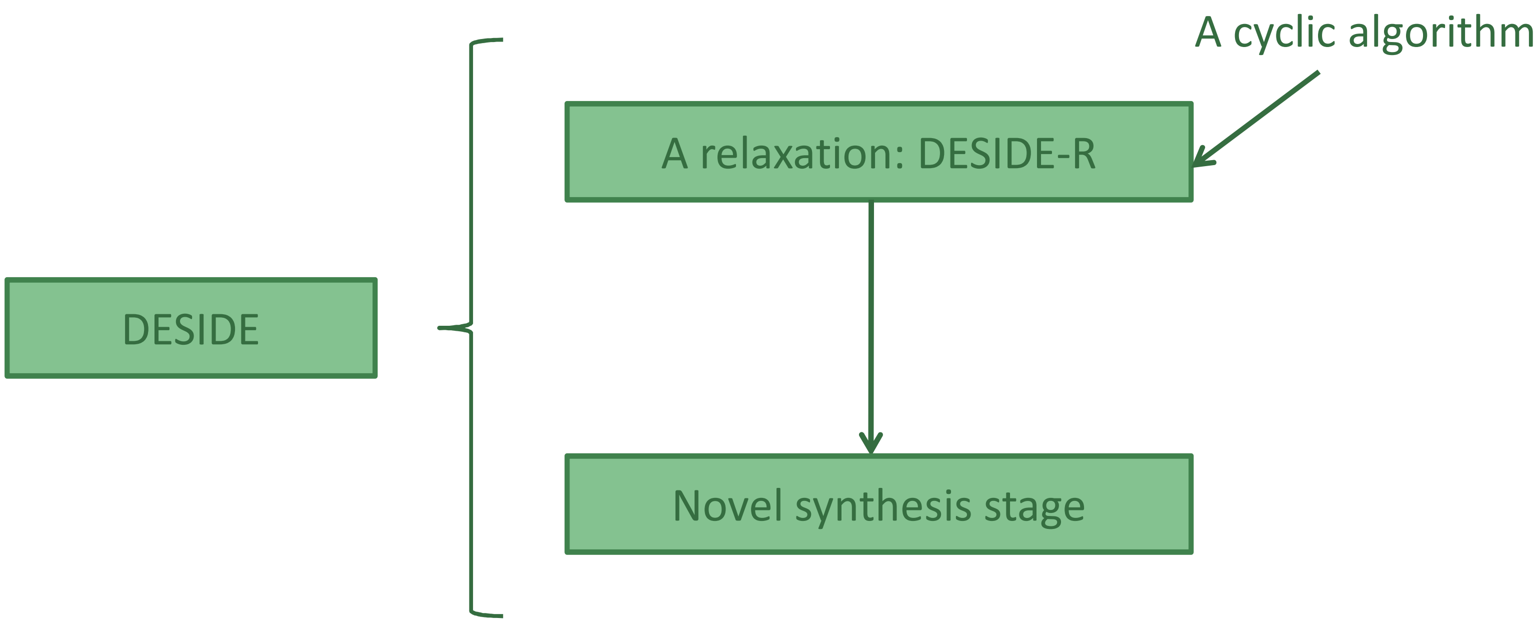
Belongs to a class of NP-Hard problems

A radar example

- A joint work from 2013
  - IUT
  - Prof. P. Stoica, Sweden
  - Prof. A. D Maio, Italy
  - Published as a book chapter in 2019 (by IET)
- Milestones of the devised method

A radar example

- The proposed method: **DESIDE** ✓
  - DopplEr robuSt joInt DEsign of transmit sequence and receive filter



A radar example

- Step 1: reformulation via rank-1 matrices

$$\mathbf{X} = \mathbf{x}\mathbf{x}^H$$

$$\mathbf{W} = \mathbf{w}\mathbf{w}^H$$

$$\mathcal{P}' \left\{ \begin{array}{ll} \max\limits_{\mathbf{X},\mathbf{W}} \min\limits_{\nu} & \dfrac{\mathbf{p}(\nu)^H \left(\mathbf{W} \odot \mathbf{X}^*\right) \mathbf{p}(\nu)}{\mathrm{tr}\left\{\left(\boldsymbol{\Sigma}_{\mathbf{c}}\left(\mathbf{X}\right) + \mathbf{M}\right) \mathbf{W}\right\}} \\ \text{subject to} & \mathrm{tr}\{\mathbf{X}\} = e \\ & \mathrm{tr}\{\mathbf{X}\mathbf{X}_0\} \geq \epsilon_\delta \\ & \mathrm{rank}(\mathbf{X}) = 1 \\ & \mathrm{rank}(\mathbf{W}) = 1 \\ & \mathbf{X} \succeq \mathbf{0} \\ & \mathbf{W} \succeq \mathbf{0} \\ & \nu \in \Omega \end{array} \right.$$

A radar example

- Step 2: SDR and apply alternating optimization

$$
\mathcal{P}_X \left\{
\begin{array}{ll}
\max\limits_{\mathbf{X},\tilde{t}} & \dfrac{\tilde{t}}{\operatorname{tr}\left\{\left(\boldsymbol{\Theta}_{\mathbf{c}}(\mathbf{W}) + (\frac{\beta}{e})\mathbf{I}\right)\mathbf{X}\right\}} \\
\text{subject to} & \mathbf{p}(\nu)^H \left(\mathbf{W} \odot \mathbf{X}^*\right) \mathbf{p}(\nu) \geq \tilde{t}, \quad \forall\, \nu \in \Omega \\
 & \operatorname{tr}\{\mathbf{X}\} = e \\
 & \operatorname{tr}\{\mathbf{X}\mathbf{X}_0\} \geq \epsilon_\delta \\
 & \mathbf{X} \succeq \mathbf{0}.
\end{array}
\right.
$$

Infinitely many constraints: The idea of the trigonometric polynomial/sum of squares to reformulate as finite SDP constraints

A radar example

- Step 3: Obtain Tx transmit vector from the solution to SDP: matrix $\mathbf{X} = \mathbf{x}\mathbf{x}^H$
  – novel synthesis stage via ESD approximation

- Summary of DESIDE

Step 0: Initialize $\mathbf{X}$ with $\mathbf{x}\mathbf{x}^H$ where $\mathbf{x}$ is a random vector in $\mathbb{C}^N$.

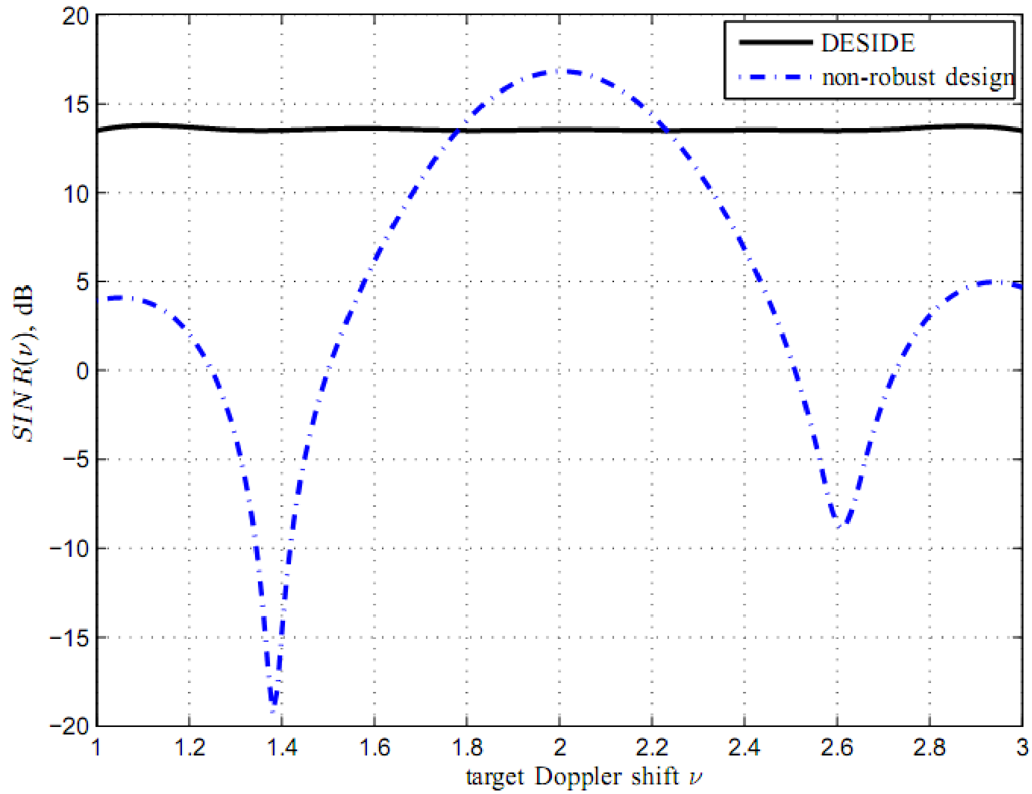
Step 1: Solve the problem $\mathcal{SDP}_W$ in (26) to obtain $\mathbf{W}$.

Step 2: Solve the problem $\mathcal{SDP}_X$ in (24) to obtain $\mathbf{X}$.

Step 3: Repeat steps 1 and 2 until a pre-defined stop criterion is satisfied, e.g. $|\min_{\nu\in\Omega} \widetilde{SINR}_{relax}(\nu)^{(\kappa+1)} - \min_{\nu\in\Omega} \widetilde{SINR}_{relax}(\nu)^{(\kappa)}| \leq \mu$ for a given $\mu > 0$.

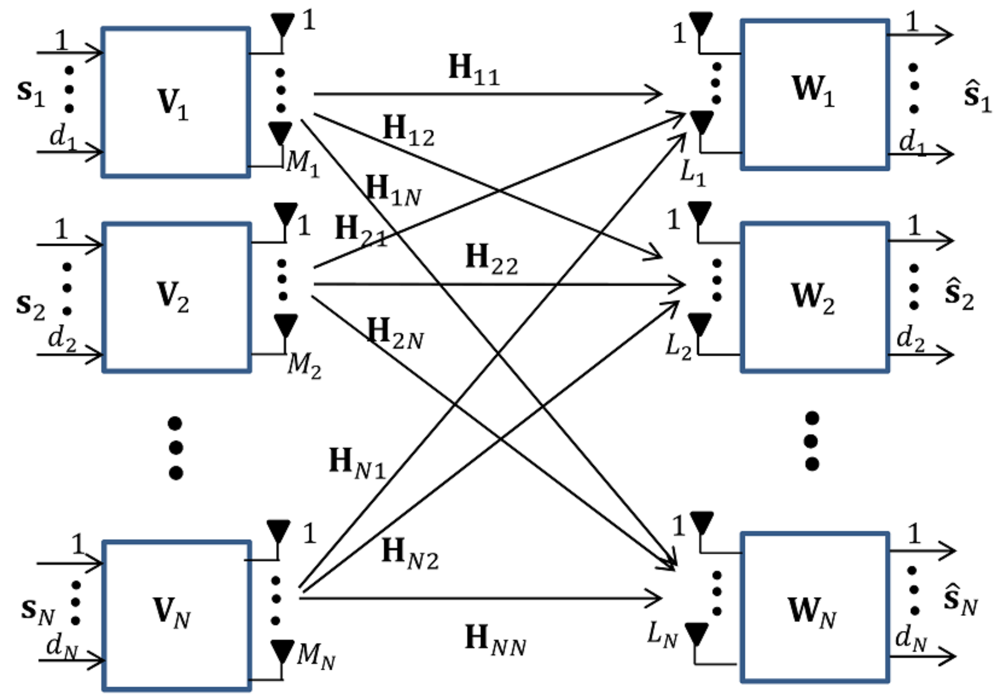
A radar example

- Numerical example: robustness



A communication example

- N pairs in MIMO-IC
  - Tx precoders
  - Rx decoders
  - Users' rates
  - Done in 2019



A communication example

- Received signal at the i-th Rx

$$\mathbf{y}_i = \underbrace{\mathbf{H}_{ii}\mathbf{d}_i}_{\text{desired signal}} + \underbrace{\sum_{j \neq i} \mathbf{H}_{ji}\mathbf{d}_j + \mathbf{n}_i}_{\text{interference plus noise}}$$

- User rate assuming MMSE decoder

$$R_i = \log\det\left(\mathbf{I}_{d_i} + \mathbf{V}_i^H \mathbf{H}_{ii}^H \left[\mathbf{\Gamma}_i + \sum_{j \neq i} \mathbf{H}_{ji}\mathbf{V}_j\mathbf{V}_j^H\mathbf{H}_{ji}^H\right]^{-1} \mathbf{H}_{ii}\mathbf{V}_i\right)$$

A communication example

- The design problem

$$\begin{aligned} \max_{\{\mathbf{Q}_i\}_{i=1}^N} \quad \min_{i=1,2,\ldots,N} \quad & R_i \\ \text{s.t.} \quad & \operatorname{tr}\{\mathbf{Q}_i\} \le p_i \qquad \forall i = 1, 2, \ldots, N \\ & \mathbf{Q}_i \succeq \mathbf{0} \qquad \forall i = 1, 2, \ldots, N \end{aligned}$$

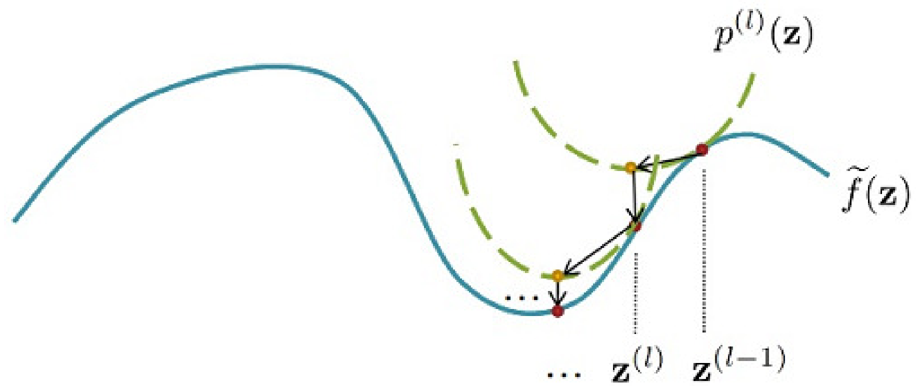
  – With Tx covariance matrix $\mathbf{Q}_i \triangleq \mathbf{V}_i \mathbf{V}_i^H$

$$R_i = \log\det\left(\mathbf{I}_{L_i} + \mathbf{H}_{ii}\mathbf{Q}_i\mathbf{H}_{ii}^H\left[\boldsymbol{\Gamma}_i + \sum_{j\neq i}\mathbf{H}_{ji}\mathbf{Q}_j\mathbf{H}_{ji}^H\right]^{-1}\right)$$

A communication example

- MM framework: Majorization-minimization
  - The problem

$$\begin{cases} \min_{\mathbf{z}} & f(\mathbf{z}) \\ \text{subject to} & c(\mathbf{z}) \le 0 \end{cases}$$



  - Majorization: a proper upper bound

$$\begin{aligned} p^{(l)}(\mathbf{z}) &\ge f(\mathbf{z}), \quad \forall \mathbf{z} \\ p^{(l)}\left(\mathbf{z}^{(l-1)}\right) &= f\left(\mathbf{z}^{(l-1)}\right) \end{aligned}$$

  - Minimization:

$$\begin{cases} \min_{\mathbf{z}} & p^{(l)}(\mathbf{z}) \\ \text{subject to} & c(\mathbf{z}) \le 0 \end{cases}$$

A communication example

- Trick: reformulation of the objective as a convex function!

$$R_i = \log\det(\mathbf{U}^H \mathbf{B}_i^{-1} \mathbf{U})$$

– with

$$\mathbf{U} \triangleq \begin{bmatrix} \mathbf{I}_{M_i} & \mathbf{0}_{M_i \times L_i} \end{bmatrix}^T$$

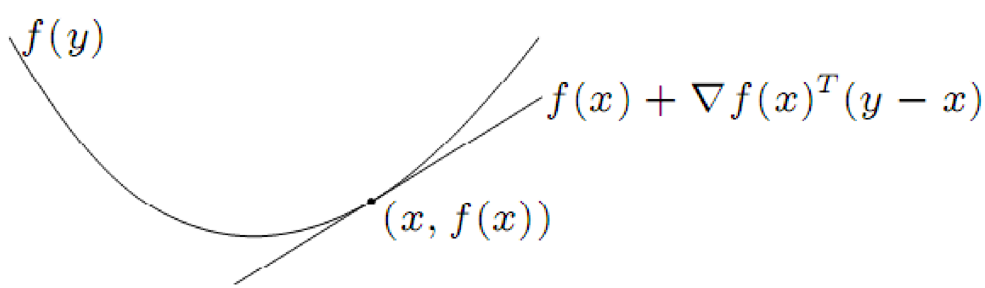
$$\mathbf{B}_i = \begin{bmatrix} \mathbf{I}_{M_i} & \widetilde{\mathbf{V}}_i^H \mathbf{H}_{ii}^H \\ \mathbf{H}_{ii} \widetilde{\mathbf{V}}_i & \mathbf{\Gamma}_i + \sum_{j=1}^{N} \mathbf{H}_{ji} \widetilde{\mathbf{V}}_j \widetilde{\mathbf{V}}_j^H \mathbf{H}_{ji}^H \end{bmatrix}$$

A communication example

- Minorize the convex function via definition of convex functions

$$\log\det(\mathbf{U}^H\mathbf{B}_i^{-1}\mathbf{U}) \geq \log\det(\mathbf{U}^H\overline{\mathbf{B}}_i^{-1}\mathbf{U}) - \mathrm{tr}\{\mathbf{F}_i(\mathbf{B}_i - \overline{\mathbf{B}}_i)\}$$

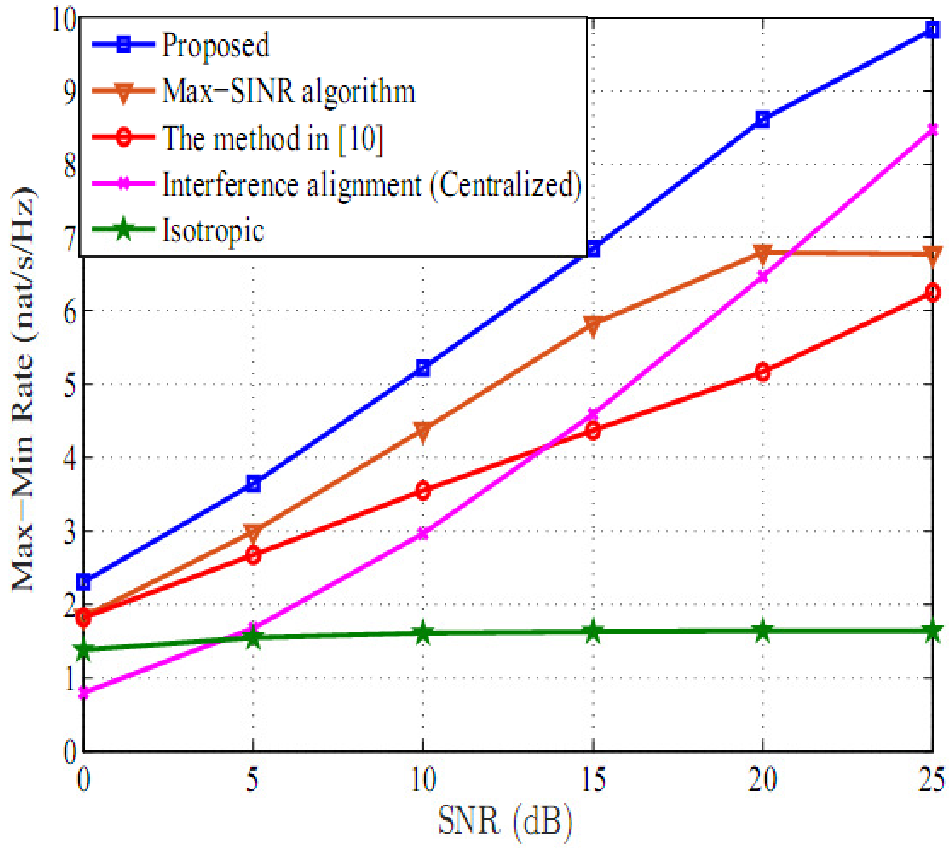
- Iterative SOCP



http://web.stanford.edu/class/ee364a/lectures.html

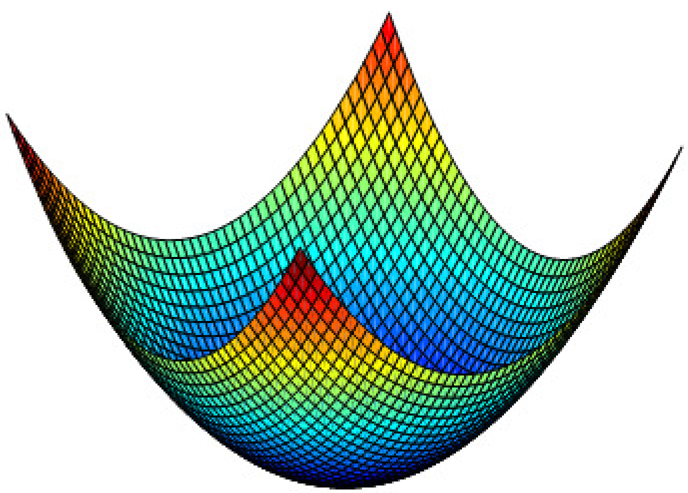
A communication example

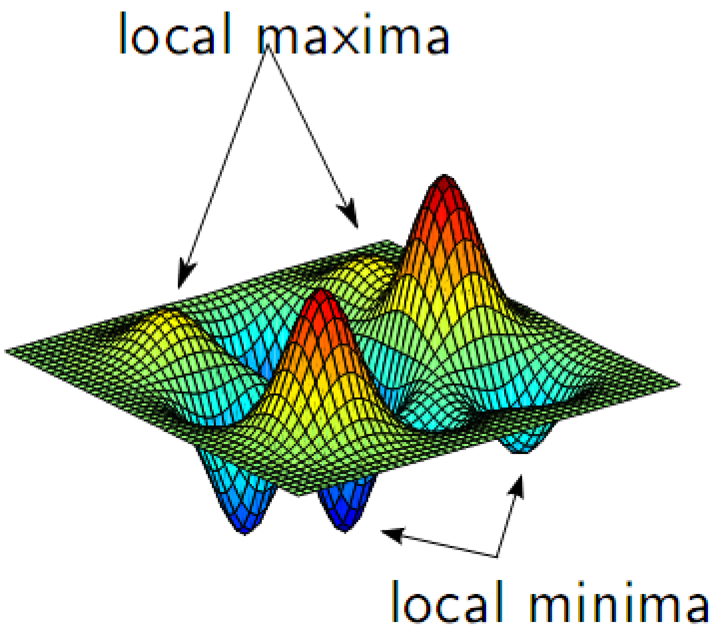
- A numerical example



A note on convergence

- Convex versus non-convex problems

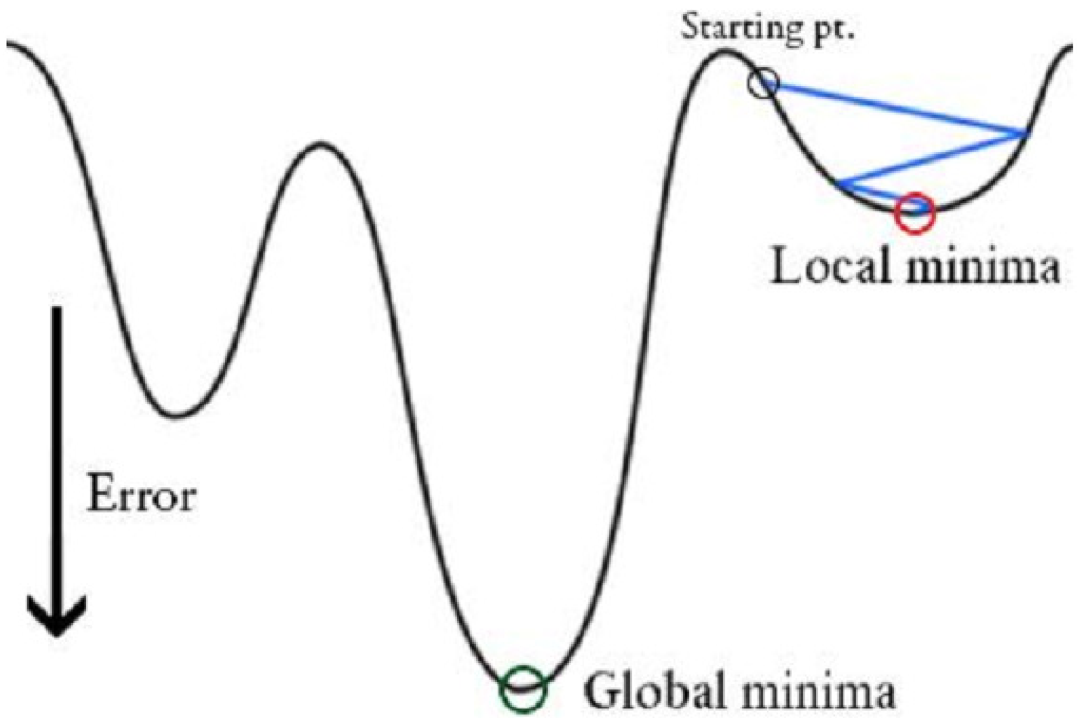




http://newport.eecs.uci.edu/anandkumar/slides/icml2016-tutorial.pdf

A note on convergence

- The effect of the starting point



https://www.cs.ubc.ca/labs/lci/mlrg/slides/non_convex_optimization.pdf

A note on convergence

- Converge to stationary points
  - Not always but under some mild conditions
  - MM, AO
  - Sequence of the objective values are always convergent
- Improvement with respect to starting point
  - Try several initializations
  - Possibility & computational burden

Summary and conclusion

- Active sensing systems: radars
  - Tx-Rx optimization: signal/filter
- Communication systems
  - Tx-Rx optimization: precoder/decoder
- Examples of radar and communication systems
  - Non-convex optimization techniques like SDR and MM